CS 360 — Introduction to the Theory of Computing
Assignment 5

University of Waterloo, Spring 2018

Due 5:00 PM, July 25, 2018.

1. A *useless state* is a state that is never entered on any input string. Show that the language

   $$L = \{\langle M \rangle \mid M \text{ is a Turing machine and has a useless state}\}$$

   is undecidable.

2. Show that the reduction relation $\leq$ is transitive.

3. Show that if $\mathbf{P} = \mathbf{NP}$, then every language $A \in \mathbf{P} \setminus \{\emptyset, \Sigma^*\}$ is $\mathbf{NP}$-complete.

4. Recall that the edit distance $d(x, y)$ of two words x and y is the fewest number of insertions, deletions, and substitutions required to transform x into y. We can extend this definition to the edit distance between two languages L_1 and L_2 by

   $$d(L_1, L_2) = \min\{d(x, y) \mid x \in L_1, y \in L_2\}.$$

   In other words, the distance between two languages is the distance between their two closest words.

   Show that if L_1 and L_2 are context-free languages, then $d(L_1, L_2)$ is uncomputable.

5. Let the class **coNP** be defined by

   $$\mathbf{coNP} = \{L \mid \overline{L} \in \mathbf{NP}\}.$$

   That is, **coNP** is the class of languages that are complements of languages in **NP**. As with many problems in computational complexity, it is not known whether $\mathbf{NP} = \mathbf{coNP}$.

   Show that $\mathbf{NP} = \mathbf{coNP}$ if and only if there exists a language L such that $L \in \mathbf{coNP}$ and L is $\mathbf{NP}$-complete.