

# CS 360 — Introduction to the Theory of Computing

## Assignment 3

University of Waterloo, Spring 2018

*Due 5:00 PM, June 15, 2018.*

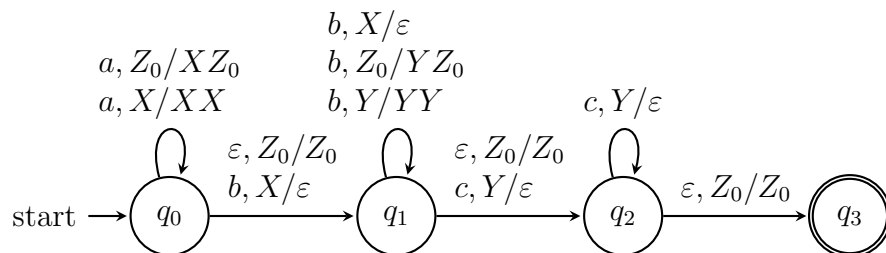
1. Consider the following context-free grammar  $G$ :

$$\begin{aligned} S &\rightarrow ABA \\ A &\rightarrow aA \mid \varepsilon \\ B &\rightarrow bB \mid \varepsilon \end{aligned}$$

- (a) Give a parse tree for the word  $abaa$ .
  - (b) Show that  $G$  is ambiguous.
  - (c) What is the language that  $G$  generates? Briefly explain how  $G$  generates this language (no formal proof necessary).
2. Consider the following context-free grammar  $G$ :

$$S \rightarrow aSa \mid bSb \mid \$$$

- (a) What is the language that  $G$  generates? Show this via induction.
  - (b) Give the equivalent grammar in Chomsky normal form.
3. Consider the following pushdown automaton  $A$ :



- (a) What language does  $A$  recognize? Briefly explain how  $A$  recognizes this language.
  - (b) Give a context-free grammar that recognizes the same language. No proof is necessary, though you may wish to give a brief explanation to help convince the reader it is correct.
4. Suppose a language  $L \subseteq \Sigma^*$  is accepted by a PDA by final state and the PDA has the property that for any word  $w \in \Sigma^*$ , the stack of the PDA on input  $w$  never has more than 3 elements of  $\Gamma$  on it. Prove that  $L$  is regular.
5. Let  $L = \{w \in \{a, b, c, d\}^* \mid |w|_a = |w|_d \wedge |w|_b = |w|_c\}$ . Show that  $L$  is not context-free.