CS 360 — Introduction to the Theory of Computing
Assignment 2

University of Waterloo, Spring 2018

Due 5:00 PM, June 1, 2018.

In this assignment, you are asked to determine whether or not the given language L is regular. To show that L is regular, provide a finite automaton or regular expression and/or argue using closure properties of regular languages. You do not need to make a formal induction argument—an informal argument that is enough to convince the marker will suffice. To show that L is not regular, provide an argument using the pumping lemma and/or closure properties of regular languages.

1. Let $L = \{a^k bw \mid w \in \{a,b\}^k, k \in \mathbb{N}\}$. Show whether or not L is regular.

2. Let $L = \{a^i b^j \mid i + j = 0 \bmod 4; i, j \in \mathbb{N}\}$. Show whether or not L is regular.

3. Let $L = \{a^m \mid m \neq n^2 \text{ for any } n \in \mathbb{N}\}$. Show whether or not L is regular.

4. Let $\Sigma = \{A, C, G, T\}$ be the alphabet of DNA bases. Let $\theta : \Sigma^* \to \Sigma^*$ be an *antimorphism*. An antimorphism is a mapping such that for $u, v \in \Sigma^*$, we have $\theta(uv) = \theta(v)\theta(u)$. We define θ by

   $$\theta(A) = T \quad \theta(C) = G \quad \theta(G) = C \quad \theta(T) = A.$$

   We call θ the *Watson-Crick involution*; given a DNA strand $w \in \Sigma^*$, $\theta(w)$ is the Watson-Crick complement of w. Observe that θ is an *involution*, which means that it has the property that for any word $w \in \Sigma^*$, we have $\theta^2(w) = \theta(\theta(w)) = w$.

   Let $L = \{w \in \Sigma^* \mid w = \theta(w)\}$. Show whether or not L is regular.

5. Let G be the following context-free grammar.

   $$S \to aSc \mid A$$
   $$A \to bA \mid \varepsilon$$

   (a) What is the language generated by G? Argue informally about how G generates this language.

   (b) Show whether or not $L(G)$ is regular.