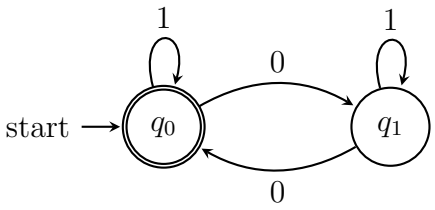
CS 360 — Assignment 1

University of Waterloo, Spring 2018

Due 5:00 PM, May 18, 2018.

1. Show for all words $u, v \in \Sigma^*$ and $k \in \mathbb{N}$, that $u(vu)^k = (uv)^k u$.

2. Consider the following DFA A:



   (a) Give the formal description for A.

   (b) What is the language that A accepts? Prove this formally.

3. Consider the language

$$L = \{w \in \{A, C, G, T\}^* \mid w \text{ ends in } AGG, CGG, GGG, \text{ or } TGG\}.$$

   Draw a state diagram for a DFA that accepts L and prove formally that your DFA accepts L.

4. Prove that if L is a regular language, the language

$$L' = \{u \in \Sigma^* \mid uv \in L \text{ for some } v \in \Sigma^*\}$$

   is also regular.

5. The *edit distance* of two words x and y is the minimum number of insertions, deletions, or substitutions of a symbol (collectively called *edit operations*) to transform x into y. For instance, we can transform the word $stars$ into the word $scarfs$ by one substitution ($t \to c$) and one insertion ($rs \to rfs$). Since two edit operations were necessary, the edit distance between $stars$ and $scarfs$ is 2. Similarly, we can transform the word $scarfs$ to $scarf$ by one deletion ($fs \to f$) for an edit distance of 1.

   Give the state diagram and formal description of an ε-NFA that recognizes the language of all words over $\{a, b\}$ with edit distance at most 1 from the word baa. Give an informal description of how and why the machine works.