

CISC 462 — Computability and Complexity

Problem Set 5

Queen's University, Winter 2017

Available March 21, 2017.

Due in class 4:30 PM, April 4, 2017.

1. What is the relationship between the following pairs of complexity classes? Possible relationships include: equal, strict inclusion in one direction (specify which direction), inclusion that is not known to be strict (based on the results we have covered). Justify your answers.
 - (a) **TIME**(4^n) and **TIME**(4^{n+4})
 - (b) **SPACE**(2^n) and **SPACE**(2^{2n})
 - (c) **NSPACE**($n \log n$) and **SPACE**($n \log^2 n$)
 - (d) **NTIME**(n^2) and **SPACE**($n^{\log n}$)
 - (e) **TIME**(n^3) and **TIME**($f(n)$), where

$$f(n) = \begin{cases} n2^n & \text{when } n \leq 10^9, \\ n^2 \log n & \text{otherwise.} \end{cases}$$

2. Consider the function $p : \Sigma^* \times \mathbb{N} \rightarrow \Sigma^* \#^*$ defined by

$$p(w, \ell) = w \#^j \quad \text{where } j = \min(0, \ell - |w|).$$

In other words, $p(w, \ell)$ adds enough $\#$ s to the end of a word w so that the length of the result is at least ℓ . For a language L and function $f : \mathbb{N} \rightarrow \mathbb{N}$, define the language $p(L, f)$ by

$$p(L, f) = \{p(w, f(|w|)) \mid w \in L\}.$$

Prove that if $L \in \mathbf{TIME}(n^6)$, then $p(L, n^2) \in \mathbf{TIME}(n^3)$.

3. Show that if $\mathbf{NP} = \mathbf{P}^{\text{SAT}}$, then $\mathbf{NP} = \mathbf{coNP}$.
4. Let UNIQUE-SAT be the problem: Given a boolean formula φ , does φ have exactly one satisfying assignment? Show that UNIQUE-SAT is in \mathbf{P}^{SAT} .
5. Show that if $\mathbf{P} = \mathbf{NP}$, then $\mathbf{P} = \mathbf{PH}$.
6. **Bonus question.** Let M be a probabilistic polynomial time Turing machine and let C be a language where for some fixed $0 < \varepsilon_1 < \varepsilon_2 < 1$,
 - $w \notin C$ implies $\Pr[M \text{ accepts } w] \leq \varepsilon_1$, and
 - $w \in C$ implies $\Pr[M \text{ accepts } w] \geq \varepsilon_2$.

Show that $C \in \mathbf{BPP}$.