CISC 462 — Computability and Complexity
Problem Set 4

Queen's University, Winter 2017

Available March 7, 2017.
Due in class 4:30 PM, March 21, 2017.

1. Let EQ_{NFA} be the language

$$EQ_{NFA} = \{\langle A, B\rangle \mid A, B \text{ are NFAs and } L(A) = L(B)\}.$$

   Show that EQ_{NFA} is in **PSPACE**.

2. Show that every regular language is in **L**.

3. Recall that A_{LBA} is the language

$$A_{LBA} = \{\langle A, w\rangle \mid A \text{ is an LBA and } w \in L(A)\}.$$

   Show that A_{LBA} is **PSPACE**-complete.

4. Let NE_{NFA} be the language

$$NE_{NFA} = \{\langle A\rangle \mid A \text{ is an NFA and } L(A) \neq \emptyset\}.$$

   Show that NE_{NFA} is **NL**-complete.

5. Recall that for a complexity class **C**, we say that a language is **C**-hard if every other language in **C** can be reduced to it in polynomial time. Show that any **PSPACE**-hard language is also **NP**-hard.

6. **Bonus question.** A *Gray code* is a sequence of code words $s_1, s_2 \ldots, s_k$ in which every code word has the same size and differs from the preceding code word by exactly one character. For example, the following is a Gray code starting with 022 and ending with 210:

$$022, 020, 120, 121, 122, 102, 100, 101, 111, 112, 110, 210$$

   Let $L = \{\langle A, u, v\rangle \mid A \text{ is a DFA where } L(M) \text{ contains each code word in a Gray code starting with } u \text{ and ending with } v\}$. Show that L is in **PSPACE**.