

# CISC 462 — Computability and Complexity

## Problem Set 3

Queen's University, Winter 2017

Available February 21, 2017.

Due in class 4:30 PM, March 7, 2017.

1. Show that  $ALL_{DFA} \in \mathbf{P}$ .
2. Show that if  $\mathbf{P} = \mathbf{NP}$ , then every language  $A \in \mathbf{P}$  except  $A = \emptyset$  and  $A = \Sigma^*$  is  $\mathbf{NP}$ -complete.
3. Let DOUBLE-SAT be the following problem: Given a boolean formula  $\varphi$ , does  $\varphi$  have at least two satisfying assignments? Show that DOUBLE-SAT is  $\mathbf{NP}$ -complete.
4. A subset of the vertices of a graph  $G$  is the *dominating set* if every other vertex of  $G$  is adjacent to some node in the subset. Then DOMINATING-SET is the following problem: Given a graph  $G$  and integer  $k$ , determine whether  $G$  has a dominating set with  $k$  nodes. Show that DOMINATING-SET is  $\mathbf{NP}$ -complete by giving a reduction from VERTEX-COVER.
5. The class *difference polynomial time*  $\mathbf{DP}$  is defined by

$$\mathbf{DP} = \{A \mid B \cap \overline{C} \text{ where } B, C \in \mathbf{NP}\}.$$

In other words,  $\mathbf{DP}$  is the class of languages that can be expressed as the intersection of a language in  $\mathbf{NP}$  and a language whose complement is in  $\mathbf{NP}$ . Let

$$L = \{\langle G_1, k_1, G_2, k_2 \rangle \mid G_1 \text{ has a } k_1\text{-clique and } G_2 \text{ doesn't have a } k_2 \text{ clique}\}.$$

Show that  $L$  is  $\mathbf{DP}$ -complete. In other words, show that  $L$  is in  $\mathbf{DP}$  and that every language in  $\mathbf{DP}$  is polynomial time reducible to  $L$ .

6. **Bonus question.** A *2cnf-formula* is a boolean formula where each clause has two literals,

$$\varphi = (a_1 \vee b_1) \wedge (a_2 \vee b_2) \wedge \cdots \wedge (a_n \vee b_n).$$

The problem 2SAT is: Given a 2cnf-formula, determine whether it has a satisfying assignment. Show that 2SAT  $\in \mathbf{P}$ .