CISC 462 — Computability and Complexity
Problem Set 2

Queen's University, Winter 2017

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Due in class 4:30 PM, February 7, 2017.

1. Consider the following languages. For each, show whether or not Rice's theorem shows that the language is undecidable.
   (a) $A = \{\langle M \rangle \mid M \text{ writes a blank symbol on the input portion of the tape}\}$
   (b) $B = \{\langle M \rangle \mid L(M) \text{ is recognizable}\}$
   (c) $C = \{\langle M \rangle \mid L(M) \text{ is finite}\}$
   (d) $D = \{\langle M \rangle \mid \overline{A_{TM}} \leq_m L(M)\}$
2. Let Σ be a finite alphabet. We denote by $FIN(\Sigma)$ the set of finite languages over Σ. That is,
   $$FIN(\Sigma) = \{L \subseteq \Sigma^* \mid L \text{ is finite}\}.$$
   Show that $FIN(\Sigma)$ is countable.
3. A *useless state* is a state that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show it is undecidable.
4. Show that the Post Correspondence Problem is decidable over a unary alphabet.
5. Let B be a language over Σ. Are the following true or false? Justify your answers.
   (a) If $B \leq_m A_{TM}$, then B is decidable.
   (b) If $B \leq_m A_{TM}$, then B is recognizable.
6. **Bonus question.** A *homomorphism* is a function $\varphi : \Sigma^* \to \Delta^*$ where Σ and Δ are alphabets. Let $w = a_1 a_2 \cdots a_n \in \Sigma^*$ and $L \subseteq \Sigma^*$ be a language. Then
   $$\varphi(w) = \varphi(a_1)\varphi(a_2)\cdots\varphi(a_n)$$
   and
   $$\varphi(L) = \{\varphi(w) \mid w \in L\}.$$
   Show that the class of decidable languages is not closed under homomorphism.