

# CISC 462 — Computability and Complexity

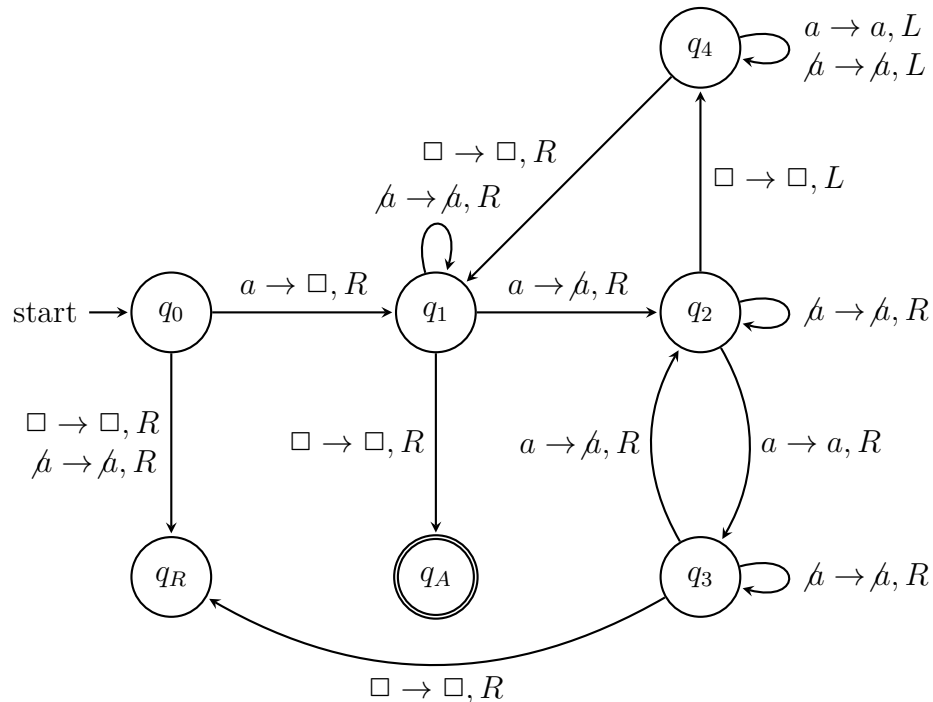
## Problem Set 1

Queen's University, Winter 2017

*Available January 10, 2017.*

*Due in class 4:30 PM, January 24, 2017.*

1. Consider the following Turing machine, which decides the language  $A = \{a^{2^n} \mid n \geq 0\}$ . The initial state is  $q_0$ , the accepting state is  $q_A$ , and the rejecting state is  $q_R$ .



Give the sequence of configurations that the machine enters when started on the input string:

- (a)  $aaa$
  - (b)  $aaaa$
2. Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet  $\{a, b\}$ .
    - (a)  $\{w \mid w \text{ contains twice as many } a\text{'s as } b\text{'s}\}$
    - (b)  $\{w \mid w \text{ does not contain twice as many } a\text{'s as } b\text{'s}\}$
  3. Show that a pushdown automaton with two stacks can simulate a Turing machine.
  4.
    - (a) Show that the class of decidable languages is closed under intersection.
    - (b) Show that the class of decidable languages is closed under complementation.

- (c) Show that the class of recognizable languages is closed under intersection.
  - (d) Show that the class of recognizable languages is closed under concatenation.
5. Let  $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ . Show that  $ALL_{DFA}$  is decidable.
6. **Bonus question.** A *Turing machine with doubly infinite tape* is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward.

Show that this type of Turing machine recognizes the class of Turing-recognizable languages.